gested by the number of vessels lately lost in that vicinity, owing to the fact that they were out in their reckoning. The bottles, which are of gutta percha, are to be sealed and thrown into the sea by passing vessels, each one containing a label showing the date and the position at which it was cast adrift. They are then supposed to drift ashore and to be recovered. The expense involved is considerable. On the bottle it is stated that a reward of five francs will be paid for the return to any of His Majesty's consuls—an instance of liberality of expenditure in the acquisition of knowledge which is almost unprecedented.

SUMMER MEETING OF THE AMERICAN FORESTRY ASSOCIATION.

By Prof. ALFRED J. HENRY, U. S. Weather Bureau.

The American Forestry Association held its summer meeting at Lansing, Mich., August 27-28, 1902, under the joint auspices of the Michigan Forestry Commission and the Michigan Agricultural College. The sessions were held in the State Capitol and the Botanical Laboratory of the Agricultural College,·Hon. Charles W. Garfield, Vice President of the Association for Michigan, in the chair.

The papers read and discussed at the meeting were for the most part upon practical problems in forestry and forest management, particularly as applied to the conditions which obtain in Michigan. It is gratifying to note in this connection that the people of that State, and indeed those of other States as well, are fully alive to the great necessity of taking prompt action looking to the preservation of their rapidly disappearing forests.

The advanced position that Michigan has taken in industrial affairs during recent years and the development of new industries has drawn rather heavily upon her water resources. The question of the constancy of stream flow and the possibility of developing additional power is now receiving attention so that a very substantial as well as a sentimental interest attaches to the preservation of the forests on the headwaters of her principal rivers.

During the last thirty-five years vast tracts of Michigan pine lands have been cut over and the merchantable timber removed. In many districts the lumberman has been succeeded by the agriculturist, and prosperous farming communities have been established. In other districts, especially in the region northwest of Saginaw Bay, the attempt at farming has not been as successful as might be wished. Many tracts of land from which the lumber has been removed were abandoned, and in course of time reverted to the State.

From the lands thus acquired the State has set apart about 57,000 acres in Roscommon, Crawford, and Oscoda counties as a forest preserve. At the same time a commission was appointed to have charge, not only of the forest preserve, but also of all matters relating to forests and forest management wherein the State was an interested party. Naturally much of the discussion of the meeting turned upon the measures best adapted to the reclamation of the waste lands, pine barrens as they are locally known, in the forest preserve and elsewhere in the State. These lands are for the most part unfit for agricultural purposes. The soil is sandy, coarse in texture, so coarse in fact that its capillary power is exceedingly low. The rain that falls upon it soon passes far below the roots of the scanty flora that now subsists upon it and is lost so far as plant life is concerned. That such a condition is not of recent origin is clearly shown by the fact that the present flora of the region is composed largely of species which have developed structural forms adapted to much less humid regions. On the other hand it should be remembered that a great part of these abandoned lands was once covered by a growth of magnificent white and Norway pine. The important question is therefore "Can not these trees be grown again?"

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The consensus of opinion as expressed at the meeting was in the affirmative, but on certain of the poorer lands it would be necessary to first plant trees of a relatively low order in the economy of nature, as for example, the jack pine, a tree that will grow on lands that have been fire-swept and abandoned by other forest trees, or left to waste by the farmer.

The forest, as was pointed out by Dr. Gifford, performs simultaneously two important functions, soil fixation and soil betterment. The improvement of the soil would be a comparatively slow process, yet with the gradual formation of humus and with the added protection of the trees the moisture conditions would also improve, especially as regards the conservation of the snowfall, much of which is now wasted. Thus the way would be paved for the return of the better species of trees.

Mr. Thomas H. Sherrard of the Bureau of Forestry, United States Department of Agriculture, gave a general description of the physical characteristics of the lands in the forest preserve. He classed the existing forest covering as (1) Swamp; (2) Jack pine plain; (3) Oak flat; (4) Oak ridge, and (5) hardwood lands, and showed the distribution of these types in a representative township. Mr. Sherrard also gave an estimate of the possible production of a second crop of timber on these lands based upon measurements of existing second growth.

The climatologist will be interested chiefly in the deliberations of the several sessions respecting the destruction of the forests, the blighting effect of forest fires, and the diminution of stream flow due to these causes. Fortunately for the State, the scars made upon her surface are not so deep or lasting as they might have been under different conditions as to climate and topography. The rainfall is generally abundant for all needs, though not heavy enough to cut and seam the surfaces from which the timber has been removed. Then, too, owing to the humid climate, the original forest has in many cases become covered with a second growth of native trees or underbrush, thus preserving the character of the original covering. So far as can be judged from the scanty data available, deforestation has not changed the climate to an appreciable degree.

THE PERMANENCY OF PLANETARY ATMOSPHERES, ACCORDING TO THE KINETIC THEORY OF GASES.

By S. R. COOK, Case School of Applied Science, Cleveland, Ohio, dated September 3, 1902.

1. HISTORICAL.

Since the development of the kinetic theory by Clausius, Meyer, and Maxwell, and especially since it has been shown by Maxwell and Boltzmann that the molecules of any gas may have velocities ranging from zero to infinity, it has been a problem of intense interest to many scientists to determine the probability that the molecules of highest velocity may escape from the outer limits of an atmosphere, and hence deduce the condition of atmospheric permanence.

The vast extent of the gaseous envelope of the sun, the absence of an atmosphere around the moon, the extent and permanency of the atmosphere of the earth and the probable existence of atmospheres on the planets are problems that arouse and hold the interest alike of astronomers and physicists.

According to the nebular hypothesis, these bodies at one time all belonged to the same nebulous mass. It may then very naturally be assumed that under similar [temperature] conditions they would each contain the same forms of matter in their atmospheres. Various hypotheses, both chemical and physical, have been presented to explain the absence of all free gases from the surface of the moon. The presence of certain markings on Mars, that appeared to be accounted for by atmospheric conditions, has caused much interesting speculation and scientific discussion as to the probable constitution of this planet's atmosphere. The existence at times of what

appear to be rapidly dissolving snow fields has been cited as evidence that the atmosphere of Mars contains water vapor.

The permanence of the earth's atmosphere according to the kinetic hypothesis was probably first discussed by J. J. Waterston in a paper on The Physics of Media, read before the Royal Society in 1846. This memoir remained in the archives of the society until discovered by Lord Rayleigh and published in the Philosophical Transactions in 1892. The publication of Waterston's paper may or may not have had a stimulating influence on the scientific thought and the minds of those who were studying problems relative to the atmosphere, but at least there seems to have been a marked awakening contemporaneous with or shortly after that event. Between the date of its submission to the Royal Society and that of its publication the kinetic theory had received notable additions from the pens of Maxwell and Boltzmann; the law of the distribution of velocities had been established, and the kinetic theory had been placed on a mathematical basis so that the limitations of Waterston's paper can now be readily seen and the problem can be discussed anew from the point of view of the more recently developed theories.

Dr. G. Johnston Stoney had, prior to the publication of Mr. Waterston's paper, been a close student of the kinetic theory of atmospheres. On December 19, 1870, he delivered a discourse before the Royal Dublin Society on the absence of an atmosphere from the moon. In this address he showed that since the potential of gravitation on the moon is such that a free molecule moving in any outward direction with a velocity of 238 meters per second will pass beyond the radius of influence of the satellite, therefore any atmosphere whose molecules are capable of occasionally reaching that velocity can not be retained by the moon. Later Dr. Stoney communicated a second paper to the Royal Dublin Society showing that if the moon once had an atmosphere composed of gases similar to those in the earth's atmosphere and had lost it by molecular diffusion, then it follows that the earth itself could not retain free hydrogen in its atmosphere, and that probably water vapor could not be retained by Mars.

The year following the publication of Mr. Waterston's paper Dr. G. H. Bryan read a paper before the British Association on the Kinetic Theory of Atmospheres, an abstract of which was published in its Nottingham Report for 1893.[1]

In 1897 Dr. Stoney collected his several papers on the subject and published his memoir on "Atmospheres upon planets and satellites" in the Transactions of the Royal Dublin Society and in the Astrophysical Journal, 1898, Vol. VII, pp. 25–55. In this memoir Dr. Stoney based his calculations on the then supposed fact that helium, although continually escaping from springs and other natural sources into the atmosphere, did not accumulate as an important constituent element of the atmosphere. If helium, having a molecular weight of two times that of hydrogen, can not be retained by the earth, but filters outward through the atmosphere and, owing to its very great velocity, overcomes the gravitational force and escapes into the void beyond, then a definite limiting ratio between the molecular weight or the mean square velocity of the molecule and the potential of gravitation can be established that expresses the conditions under which the molecules of any gas can not be retained by a planet.

The writer, when reporting Dr. Stoney's memoir before the Physics Colloquium in the University of Nebraska, was impressed with the very great importance of this ratio in relation to the kinetic theory of atmospheres, provided it could be raised to the rank of a mathematical deduction under the laws of the kinetic theory by the application of the Boltzmann-Maxwell law of distribution of molecular velocities. He accordingly attempted to verify Dr. Stoney's results by applying the well-

established laws of the kinetic theory to the problem, and his paper "On the escape of gases from planetary atmospheres according to the kinetic theory" appeared in the Astrophysical Journal for January, 1900. Contemporaneous with the publication of this paper Dr. G. H. Bryan read a paper on the "Kinetic theory of planetary atmospheres" before the Royal Society. This paper appeared in the Transactions of the Royal Society, London, 1901.

2. WATERSTON'S METHOD.

Waterston based his calculations of the permanancy of an atmosphere on the assumption that the atmosphere was composed of molecules whose velocities at any position in a vertical column of the atmosphere are all the same and are proportional to the speed a molecule would attain in falling from the limits of the atmosphere to that position. The height of an atmosphere is thus proportional to the velocity of the molecule at the surface of the planet. In making this assumption, Waterston was in advance of his time with respect to the kinetic theory. He was, indeed, one of the first to apply mathematics to its more simple conceptions.

Since Waterston takes the velocity, v, of the molecules proportional to the height of the atmosphere, he concludes that, if the molecules were all moving in vertical lines, the height of an atmosphere would be the distance the molecule would go in overcoming gravity, or

1.
$$H = \frac{v^2}{2g},$$

but by considering the effect of the molecules moving at all possible angles to the vertical, he obtains

2.
$$H = \frac{v^2}{g}$$

as the height of an atmosphere, on the assumption that gravity is constant throughout the whole of this height. When the variation of g and the specific gravity of the gas composing the atmosphere are considered, Waterston obtains

3.
$$H = \frac{Rv^2}{Rgs - v^2},$$

where R is the radius of the earth and s is the specific gravity of the gas, air being taken as unity.

If under these conditions an atmosphere is escaping, H becomes infinity and putting $H =$ infinity in equation 3 we find,

4.
$$\frac{v^2}{s} = Rg.$$

Since for any gas the square of the velocity is directly proportional to the absolute temperature, and assuming that all molecules of the same gas, at a distance r from the center of the planet, have equal velocities, Waterston finds the temperature of the earth at which an atmosphere of air would slowly evaporate into space to be 65,760° F. Again, the temperature at which the moon would loose its atmosphere, taking into consideration the attraction of the earth, would be 2,405° F. From these conclusions it can easily be deduced that the atmospheres of the planets would be permanent even at much higher temperatures than is usually assumed for nonluminous bodies.

Waterston's results as to the permanency of atmospheres are of historic interest only. If, however, he had known and made use of the laws of distribution of velocities developed later by Maxwell his results would have been of much scientific value.

3. RESEARCHES BY DR. G. JOHNSTON STONEY.

As already mentioned, Dr. Stoney based his calculations on the permanency of atmospheres on the supposition that helium was escaping from the earth's atmosphere. Helium is continu-

[1] B. A. A. S. Nottingham Report, p. 682, 1893.

ally being supplied to the atmosphere from springs and other natural sources, and since the inertness of this gas makes it highly improbable that it enters into combination with other constituent elements of the atmosphere, therefore it must either remain a constituent element, or be absorbed, or escape.

Dr. Stoney argued that since it did not become a constituent element in the atmosphere it must be escaping from its outer limits, "Indeed, so promptly escaping," to quote his own words, "that the amount in transitu is too small for the appliances of the chemist to detect it." On the other hand water vapor, with a molecular weight of nine times that of hydrogen, is not sensibly leaving the earth's atmosphere. Hence Dr. Stoney concludes that the boundary between those gases that can effectually escape and those that can not, lies somewhere between gases consisting of molecules with twice the atomic weight of hydrogen, and gases consisting of molecules whose mass is nine times that of hydrogen.

Having given the velocity of the mean square[2] of the molecules of air at 0° C., we are able to calculate the velocity of the mean square of the molecules of any other gas whose density (compared with air at 0° C.) is ρ, by the formula of Clausius, as follows:

5. $W = 485 \sqrt{\dfrac{T}{273\,\rho}}$ met./sec.,

where T is the absolute temperature, and 485 is the velocity of the mean square for air at 0° C. Using hydrogen as our standard this equation transforms into

6. $W = 111.4 \sqrt{\dfrac{T}{\rho}}$ met./sec.,

where ρ' is the density of the gas considering hydrogen as unity.

In order to determine the velocity that a small body will attain in falling from infinity to the surface of a planet or other attracting body B, we have to consider the dynamical equation for acceleration and potential. Assuming that each body is homogeneous and spherical the acceleration at the surface of B, whose mass is M and radius R, is

7. $a = \dfrac{M}{R^2}$

and the potential of gravity at the surface is

8. $K = \dfrac{M}{R}.$

But K expresses the kinetic energy stored up in unit mass by a body in falling upon the surface of B from infinity, hence,

9. $K = \tfrac{1}{2} v^2$

where v is the velocity that would be acquired in falling from infinity.

If a' is the total acceleration including the acceleration γ, due to the rotation of the earth at the equator, then

10. $a' = g + \gamma,$

and if u is the velocity of the earth's surface at the equator and R its radius, then

11. $\gamma = \dfrac{u^2}{R}.$

If K' is the potential at a distance h from the surface of B whose radius is R, then

12. $K' = \dfrac{R^2}{R+h}\,a.$

Substituting the values 6,378 km., 200 km. 978.1 cm./sec.[2] and 464 m./sec. for the values R, h, g, and u at the equator of the

earth, equation 9 gives for the least velocity a small body must have in order that it may go to infinity from a position 200 km. from the surface of the earth,

                    $v' = 1101500$ cm./sec. $= 11.015$ km./sec.

Deducting the equatorial velocity of the surface of the earth, (0.478 km./sec.), there results $v' - u = 10.537$ km./sec, or allowing for prevailing westerly winds one may take 10.5 km./ sec. $= 10,500$ met./sec. as the least possible velocity that a molecule, favorably situated, must have in order that it may pass beyond the earth's attraction. This velocity $(v' - u)$ may be conveniently designated as the critical velocity.

The temperature at a position 200 km. from the surface of the earth was taken as $-66°$ C., or 207° absolute.[3] The velocity of the mean square for hydrogen at this temperature, by equation 6, is 1,603 met./sec., for helium 1,133 met./sec., and for water vapor 534 met./sec. The ratios of the above critical velocity to these velocities of the mean square or v'/w are for hydrogen 6.55, for helium 9.27, and for water vapor 19.66.

Now, since helium and hydrogen are both assumed to escape from the atmosphere of the earth while water vapor does not escape appreciably, Dr. Stoney concludes that any gas having a ratio less than 9.27 will escape, but gases whose ratio is equal to or greater than 19.66 will be imprisoned by the earth in its atmosphere. The following table is a summary of the computations and results arrived at by Dr. Stoney for all the planets and for an assumed temperature of $-66°$ C. or 207° absolute:

TABLE 1.

Name of planet or satellite.	Critical velocity in meters, v'.	Velocity of the mean square in meters, w.	Density of a gas that will escape as freely as does helium from the earth, ρ'.	Density of gas that will escape as freely as hydrogen ρ''.	Lightest of the known gases or vapors that will not escape.	Molecular weight.
Moon	2380	257.	19.5	39.	Carbon dioxide	44
Mercury	4641*	500.6	10.25		Nitrogen	28
Venus	9546	1029.	2.56†		Water vapor	18
Earth	10500	1133.	2.		Water vapor	18
Mars	4803	517.	9.57		Nitrogen	28
Jupiter	47233	5095.	0.099		Hydrogen	2
Saturn	24508	2633.		0.37	Hydrogen	2
Uranus	17299	1865.		0.74	Hydrogen	2
Neptune	18002	1942.		0.68	Hydrogen	2

* Assuming its rotation period to be 88 days.   † Calculated by the writer.

From the foregoing results deduced on the supposition that helium escapes from the earth, it follows that the moon can not retain an atmosphere whose molecules are less than 19.5 times the mass of the molecules of helium, or 39 times that of hydrogen. Mercury can not imprison an atmosphere whose molecules were less than 10.25 times the mass of the molecules of helium. Venus can retain an atmosphere similar to that of the earth, while Mars can not imprison water vapor at a temperature greater than $-78.3°$ C. The planets Jupiter, Saturn, Uranus, and Neptune, can all retain an atmosphere whose molecules are less than the molecules of hydrogen.

4. PERMANENCY OF ATMOSPHERES ACCORDING TO THE KINETIC THEORY.[4]

These results attained by Dr. Stoney are so important and so essential to the explanation of the permanency of atmospheres on the earth and planets that it seemed to me that the important factors in the determination of atmospheric permanency should, if possible, be placed on a rigorous mathematical basis. Being convinced that the well-known laws of the kinetic theory, and especially the Boltzmann-Maxwell law of distribution of velocities, around the velocity of the mean square from zero to infinity, would apply to the problem. I

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[2] The velocity W, whose square is the mean of the squares of the velocity, v, of the individual molecules, n, of a gas, is, for brevity, called "the velocity of mean square," and is expressed algebraically by the formula $W = \sqrt{\dfrac{\Sigma v^2}{n}}$ [ED.]

[3] This temperature has been exceeded at the surface of the earth, and hence is not probable at 200 km. from the surface.

[4] Astrophysical Journal, January, 1900.

have attempted to verify Dr. Stoney's results by the application of the Boltzmann-Maxwell law, assuming such conditions as to boundaries and temperature as would make the number of escaping molecules a maximum. Since it is not possible in the present state of our knowledge to determine the temperature, density, or velocity of the molecules at the limit of the earth's atmosphere, or even to fix a definite boundary to its atmosphere, it becomes necessary to resort to some special hypothesis as to boundary and temperature. Even if the factors above referred to were known for a complex atmosphere composed of a number of different gases, yet the density of the hydrogen or helium atmosphere, even at the surface of the earth and at 0° C. are not accurately determined quantities. The method resorted to therefore was to assume a hydrogen or helium atmosphere containing the same number of molecules as is contained in the actual complex atmosphere, and to assume for each of these atmospheres arbitrary boundaries, the temperatures of which could be closely approximated.

This method has the advantage of eliminating the question as to the applicability of the kinetic theory to the limits of the atmosphere, where the paths of the molecules do not conform to straight lines; it also had the advantage of treating the atmosphere as a simple gas. In order that there may be no doubt as to the application of the method, and in order that my results may be compared with those obtained by Dr. Stoney four conditions were assumed as to the hydrogen or helium atmospheres, viz, (1) that the atmosphere is a spherical layer at the surface of the earth, at a mean temperature of 5° C., and whose thickness is the mean free path of a molecule having the critical velocity; (2) that the atmosphere is 200 km. in height, whose outer layer is at — 66° C., according to Dr. Stoney[5]; (3), that the atmosphere is 20 km. in height and the temperature of its outer limits — 66° C., in accordance with the recent balloon ascensions at Paris and Berlin; (4) that the atmosphere is 50 km. in height, and its outer layer at — 180° C. according to Ferrel. These atmospheres of hydrogen or helium were assumed to have at the respective limiting heights well-defined boundaries, beyond which if a molecule passed it could not be returned to the atmosphere by impact.

The number of molecules per unit volume at the boundary of these atmospheres was assumed to be the same as would be contained in unit volumes of the earth's atmosphere at the respective heights. Molecules of hydrogen or helium in their respective atmospheres will not be in a position to escape, no matter what their velocity, unless they are at some time within a distance equal to their free paths from the limits of the atmosphere. It is, therefore, only necessary to consider the molecules contained in a spherical shell at the boundaries of their respective atmospheres, and whose thickness is λ', where λ' is the mean free path of the molecules having the critical velocity.

The number of molecules in this spherical layer was computed from the well known experimental formula for pressure,

13. $P_h = p_o e^{-\frac{h}{H}}.$

The number of molecules per unit volume at normal pressure is taken as 10^{19} according to Maxwell's determination.

Having any given number of molecules, N_o, whose velocity of the mean square is c_o, it is a very simple matter to calculate the number of molecules, dN_o, that will have any given velocity, c_1; this number dN_o is, according to Maxwell, given by the differential equation:

14. $dN_o = \dfrac{4N_o}{C_o \sqrt{\pi}} \times \dfrac{C^2}{C_o^2} \times e^{-C^2/C_o^2} \, dC$

Integrating this equation between the limits $C = c'$ and $C = \infty$

⁵ The temperature at 20 km. is probably nearer — 74° C.

where c' is the critical velocity, we find the number of molecules n' that will have a velocity rc_o when $r =$ or > 1 as given by the equation,

15. $n' = 2\, n_o\, e^{-x^2} \left(x + \frac{1}{2x} - \frac{1}{4x^3} + \frac{3}{8x^5} - \frac{15}{16x^7} + \ldots\right)$

where x is written for $\dfrac{2r}{\sqrt{\pi}}$. When r is a quantity much greater than unity this series becomes rapidly converging, and n' for any velocity rc_o can be readily evaluated. In the calculations that follow r varies from 5.92 for hydrogen at 5° C. to 14.5 for helium at —180° C., and only the first three or four terms of the series need be employed.

Having then determined the number of molecules, N_o, in any of the foregoing spherical shells whose thickness is λ, and by the above formula having determined the number of molecules, n', that will have a velocity equal to or greater than the critical velocity, it is only necessary to determine the probability that these n' molecules will be so favorably situated as to be emitted by the spherical shell into the void beyond.

If n is the number of molecules in the spherical shell at the boundary of the respective atmospheres and n' is the number of molecules among the n molecules that have a velocity equal to or greater than the critical velocity, then in order that the n' molecules be so favorably situated as to be emitted by the spherical shell through its outer surface, they must have positive component velocities normal to the surface of the spherical shell equal to the critical velocity (considering velocities away from the earth as positive). If n_1 be the number of molecules in unit volume in the spherical shell, R_1 its radius, and $r\lambda = \lambda'$ its thickness, then

16. $n' = 4\pi R_1^2 r\lambda n_1,$

where n' is the number of molecules in the spherical shell.

Since the critical velocity, c', is equally probable in all directions, in order to find the number of molecules that will pass through the outer surface of the spherical shell with a velocity c' or greater we determine c'_m the mean velocity of the molecules having a velocity between the critical velocity and infinity. Since any molecule will escape whose component velocity normal to the shell is

17. $c_1 = c'_m \cos \theta,$

the proportion of those that will escape during the time t_1 is $\varphi / 4\pi$; where t_1 is the time of the mean free path $r\lambda$ of those molecules, and φ is the solid angle of aperture 2θ. Hence, to the first order of approximation the number of molecules that will escape in time t_1 will be:

18. $n_2 = 2\pi R_1^2\, r\lambda K n'(1 - \cos \theta),$

where K is written for $2e^{-x^2}\left(x + \frac{1}{2x} - \frac{1}{4x^3} + \ldots\right)$ the term that occurs in equation 15. The number that will escape in any other time, T, will be

19. $n_3 = 2\pi R^2 r\lambda K n'(1 - \cos \theta)\, \dfrac{T}{t_1}.$

This formula gives a maximum limit to the number of molecules that will be emitted from the spherical shell with a positive component velocity normal to the surface equal to or greater than the critical velocity.

The quantities on the right hand side of equation 19 can all be determined from the kinetic theory, except the critical velocity c', which, by combining equations 9 and 12, is given by the following:

20. $c'^2 = 2a\dfrac{R}{R_1},$

a being the total acceleration, R the radius of the earth, and R_1 the radius of the spherical shell.

In the following tables the first three columns specify the

four conditions described at the beginning of this section. The fourth column gives the critical velocity C' in kilometers per second. The fifth column gives r, or the ratio of the critical velocity to the mean velocity. The sixth column gives N_3, or the number of molecules that will escape during the time t_1. The seventh column gives the number of molecules that will escape in one year. And the eighth column gives the number of molecules that would have escaped during the period of the earth's existence, supposing that to be 10^7 years, in accordance with Lord Kelvin's latest figures.

TABLE 2.—*Hydrogen.*

Conditions.	Temperature. °C	Height in km.	Critical velocity, C'.	Critical velocity divided by mean velocity, r.	N_3 (molecules in c.c.) $T=t_1$ sec.	N_3 in c.c. $T=1$ year.	N_3 in c.c. $T=10^7$ years.
No.	°C						
1	5	λ'	11.	5.92	80.24×10^{19}	$33.04 \times 10^{6}_{-7}$	33.04×10^{15}
2	−66	200	10.5	6.55	45.15×10^{9}	23.58×10^{-2}	54.28×10^{5}
3	−66	20	10.98	6.85	72.61×10^{-4}	54.28×10^{-16}	54.28×10^{-6}
4	−180	50	10.90	10.14	66.8×10^{-4}	43.5×10^{-16}	43.5×10^{-8}

TABLE 3.—*Helium.*

	°C						
1	5	λ'	11.	8.37	19.8	10.34×10^{-11}	10.34×10^{-4}
2	−66	200	10.5	9.27	44.5×10^{-18}	22.10×10^{-30}	22.10×10^{-23}
3	−60	20	10.98	9.78	50.15×10^{-33}	26.73×10^{-80}	26.73×10^{-86}
4	−180	50	10.90	14.5	19.27×10^{-33}	91.6×10^{-93}	91.6×10^{-86}

The values of N_3 were computed from equation 19 and show the number of molecules or cubic centimeters of hydrogen that would escape from a hydrogen or helium atmosphere under the specified conditions.

Condition No. 4 represents most nearly the condition of the outer limits of the earth's atmosphere: 43.5×10^{-8} or less than one billioneth of a cubic centimeter of hydrogen would escape from a hydrogen atmosphere thus bounded and conditioned in 10,000,000 years, and 91.6×10^{-86} c. c. of helium would escape from a helium atmosphere similarly conditioned, during the same period. Since only a very small part of a cubic centimeter of hydrogen will escape from a hydrogen atmosphere during the possible age of the earth, it is evident that the amount which is actually escaping from the very attenuated hydrogen atmosphere that no doubt exists in the upper strata of the earth's atmosphere is insignificant. The amount of helium escaping would be zero.

There are approximately 10^{24} c. c. of air in the earth's atmosphere and under the most favorable conditions less than 10^{10} c. c. of hydrogen would escape from an atmosphere of hydrogen whose outer layer was 5° C. and whose density was the density of hydrogen at atmospheric pressure, during one year. It would under these conditions take 10^{14} years for an amount of hydrogen equal to the earth's atmosphere to escape. Under the most favorable conditions it would take 10^{11} years for one c. c. of helium to leave the earth, while under the conditions assumed by Dr. Stoney it would take 10^{31} years for 22 c. c. of helium to escape.

Let us now apply the results attained for an atmosphere of hydrogen on the earth to the atmospheres of the moon and planets. The relation between the velocities of the molecules and the absolute temperature is,

21.
$$\frac{C'^2}{C_0^2} = \frac{T}{T_0}$$

The following table gives the temperature centigrade of the outer layer of a planet or moon that would enable its atmosphere to escape at the same rate that hydrogen would escape from an atmosphere of hydrogen whose outer layer is conditioned as in (1) and whose critical velocity is five times the velocity of the mean square at 0° C., the number of molecules

in the hydrogen atmosphere being the same as in the earth's atmosphere and the time allowed for the atmosphere to escape being 10^9 years, or 100 times Lord Kelvin's age of the earth.

TABLE 4.

	C'	C_t	Hydrogen.		Air.		Carbondioxide.	
			t.	r_1	t.	r_1	t.	r_1
	km./sec.	km./sec.	°C		°C		°C	
Moon	2.380	.476	−256	1.24	−10	4.7	274	6.6
Mercury	4.468	.8936	−209	2.4	894	9.2	1371	12.4
Venus	9.540	1.909	20.5	5.185	5031	19.27	7403	26.5
Mars	4.803	.960	−195	2.66	1139	9.9	1807	13.3
Earth	10.5	2.100	291	5.7	9937	21.7	14447	29.17

C' is the critical velocity in kilometers per second; C_t is the velocity of the mean square $= C'/5$; t is the temperature of the outer layer of the atmosphere in centigrade degrees, and r_1 is the ratio of the critical velocity to the velocity of the mean square at 0° C.

It is evident, from Table 4, that an atmosphere of hydrogen would escape from the moon at a temperature of −256° C., an atmosphere of air at −10°, and an atmosphere of carbondioxide at 274° C. On all the superior planets and at 0° C. an atmosphere of air and carbondioxide would be permanent.

5. THE KINETIC THEORY OF PLANETARY ATMOSPHERES BY DR. BRYAN.

Dr. Bryan in his paper of 1901, above referred to, on the Kinetic Theory of Planetary Atmospheres extends the Boltzmann–Maxwell law of the distribution of the velocity of molecules to the condition of a planetary atmosphere in which the molecules are under an external force and have in addition to their velocity of the mean square a velocity due to the rotation of the planet.

Assuming the Boltzmann–Maxwell law of velocity distribution for a quiescent gas Dr. Bryan finds that the law for the molecular distribution in an atmosphere of a rotating planet may be expressed in the following form

22.
$$n \cdot h m \left\{ -(\xi'^2 + \eta'^2 + \zeta'^2) - V - \tfrac{1}{2}\Omega^2(\xi^2 + \eta^2) \right\} d\xi\, d\eta\, d\zeta'\, d\eta'\, d\zeta'$$

where n is proportional to the total number of molecules; ξ', η', ζ' are the component velocities in the direction ξ, η, ζ; V is the gravitation potential and Ω the angular velocity.

In this expression the distribution of coordinates and relative velocities is the same as if the axis were fixed and the potential of the field of force were increased by the term $-\frac{1}{2}\Omega(\xi^2 + \eta^2)$. This term represents the potential of centrifugal force due to rotation with angular velocity Ω.

Using this form for the function representing the frequency of distribution Dr. Bryan finds that the rate at which the molecules of any planetary atmosphere are escaping across a concentric spherical surface is given by the formula

23.
$$\frac{4\pi n}{h^3 m^3 \Omega}\left\{ e^{hm\Omega r Q}\left[1+\frac{\Omega r}{4(Q-\Omega r)}\left\{1-\frac{1}{hm(Q-\Omega r)^2}+\frac{1}{h^2 m^2(Q-\Omega r)^3}\right\}\right] + e^{-hm\Omega r Q}\left[1-\frac{\Omega r}{4(Q+\Omega r)}\times\left\{1-\frac{1}{hm(Q-\Omega r)^2}+\frac{1}{h^3 m^2(Q-\Omega r)^3}\right\}\right]\right\}$$

Where $Q^2 = 2\frac{m}{r}$. This formula might be applied to determine the rate at which a planet is losing its atmosphere. The above law of distribution, however, does not hold beyond a certain distance from the planet. In order to find the critical surface beyond which the formula (23) does not apply, Dr. Bryan de-

termines the condition for the limit of the height of an atmosphere by the following method.

According to the kinetic theory the termination of an atmosphere is determined by the fact that the density of the atmosphere is proportional to

24. $$e^{-hm}\left\{v - \tfrac{1}{4}\Omega^2 r^2\right\}$$

where $r^2 = \xi^2 + \eta^2$.

"Where $dv/dr > \Omega^2 r$ the density decreases as we proceed outwards from the axis of the planet. It becomes minimum when

25. $$dv/dr = \Omega^2 r$$

and begins to increase again outward when $dv/dr < \Omega^2 r$, hence the point at which the centrifugal force is just balanced by the planet's attraction is the point of minimum density in the atmosphere according to the above law of permanent distribution. And since the atmosphere does not extend to infinity, we conclude that it can not be permanent unless the density at the point of minimum density is infinitesimal and practically zero."

Dr. Bryan then proceeds to calculate the ratio of the density of a gas at the surface of the planet to its density at the critical surface. This ratio is called the critical density ratio. The critical velocity ratio can be very readily calculated, since according to formula (25) the minimum density at the critical surface is proportional to

26. $$e^{-hm\,(v - \frac{1}{2}\,\Omega^2\,r^2)}$$

The condition for permanency of an atmosphere requires that this ratio be very large, hence in general it will be sufficient to know this ratio correctly to the nearest power of 10 and this is shown by tabulating the logarithms of this ratio to the base 10.

If V_0 is the gravitational potential at the planet's surface; u the velocity of the planet's equator due to axial rotation; V_1 the combined potential of gravitation and centrifugal force at the critical surface, then the logarithm of the critical density ratio is equal to

27. $$\mu hm\,(V_0 + u^2/2 - V_1)$$

where μ is the modulus of the common logarithms. The data used was the same as given in Dr. Stoney's memoir[6].

Table 5 gives the calculated density ratio for hydrogen relative to the earth.

TABLE 5.

Absolute temperature.	Centigrade temperature.	Logarithm of critical density ratio.
100°	—173°	50.951
200°	— 73°	25.475
300°	27°	16.987

The meaning of this table will be made more apparent by a concrete example. Suppose with Dr. Stoney we take 10^{21} as the number of molecules in each cubic centimeter of gas at normal pressure, and normal temperature, then at the critical surface and at $+27°$ C. there will be $10^{21-16.987}$ or approximately 10^4 or 10,000 molecules per cubic centimeter. At the critical surface and at $-73°$ C. there will be $10^{21-25.475}$, or $10^{-4.475}$, or one molecule for every 30,000 cubic centimeters at the critical surface. At $-173°$ C. only one molecule will occur in 10^{30} cubic centimeters at the critical surface. It is evident that in the last two cases a hydrogen atmosphere will be quite permanent, while in the first case a considerable quantity of the hydrogen will be in a position to escape.

The calculation of this critical density ratio may easily be extended to the planets. Table 6 gives the logarithms of the ratios for hydrogen for several planets.

[6] See Table No. 1.

TABLE 6.

Planets.	100° absolute.	200° absolute.	300° absolute.
Venus	40.6360	20.3180	13.5453
Earth	50.9510	25.4750	16.987
Mars	10.4690	5.2345	3.4896
Jupiter	711.9400	355.9700	237.3100
Saturn	165.9800	82.9900	55.3300

From the magnitude of these logarithms of the critical density ratio the permanency of a hydrogen atmosphere on any of the planets, with the exception of Mars, is quite evident at temperatures below 200° absolute, and for Jupiter and Saturn at even higher temperatures.

Since the logarithm of the critical density ratio is directly proportional to the density and inversely proportional to the absolute temperature, its value may be easily determined for any other gas or any other temperature; thus the logarithms of the critical density ratio for helium on the earth and for water vapor on Mars are given by multiplying the values for hydrogen by 2 and 9, respectively.

TABLE 7.

	100° absolute.	200° absolute.	300° absolute.
Helium on the earth	101.90	50.95	33.97
Water vapor on Mars	94.22	47.11	31.41

The logarithms of the critical density ratio give a very good idea in regard to the permanency of an atmosphere on a planet, but they do not give the amount of the atmosphere that will escape from the planet in any given time. Dr. Bryan has, however, also calculated the time required for the escape of an amount represented by a layer covering the planet 1 centimeter thick. He used the formula,

28. $$E = \frac{25t\ R^2 q}{144\ .\ a^2},$$

where E is the time in years, t the number of seconds in E years, and q the mean velocity.

For hydrogen at 100° absolute temperature and for E equal to one year, the following table obtains for logarithm E:

TABLE 8.

Hydrogen at —173° C. = 100° absolute.	Log. E.
Earth	14.40133
Venus	14.35456
Mars	14.35149
Jupiter	13.47129
Saturn	13.27377

From this Dr. Bryan deduces that "If the logarithm of the critical density ratio for a given gas at a given temperature relative to a given planet is about 14, the total rate of effusion of that gas across the critical surface would be equivalent to the removal of the amount of that gas present in a layer 1 centimeter thick over the surface of the planet in a period of time comparable with a year."

The number of years in which a superficial layer of gas 1 centimeter thick will escape from a planet may also be calculated. The following table obtains for atmospheres of hydrogen and helium on the earth:

TABLE 9.

Hydrogen at absolute temperature.	Helium at absolute temperature.	Years.
100	200	3.54×10^{36}
150	300	3.06×10^{19}
200	400	8.40×10^{10}
250	500	$6.02 \times 10^5 = 602,000$
300	600	$2.22 \times 10^2 = 222$

For water vapor in Mars Table 10 obtains:

TABLE 10.

Vapor of water at absolute temperature.	Years.
200	1.22×10^{33}
250	3.37×10^{23}
300	1.94×10^{16}
400	$2.40 \times 10^{9} = 2,400,000,000$
500	$4.28 \times 10^{4} = 42,800$
600	$1.06 \times 10^{2} = 106$

Dr. Bryan arrives at the following conclusions:

1. The earth's attraction is capable, according to the kinetic theory, of retaining a gas of twice the weight of hydrogen in the form of a (practically) permanent atmosphere of uniform temperature as high as any temperature commonly existing in its present atmosphere.

2. The vapor of water is similarly capable, according to the kinetic theory, of existing on Mars in the form of a (practically) permanent atmosphere of uniform temperature at any ordinary temperature.

It appears from the foregoing that according to the kinetic theory the assumption that helium, because of its frequently recurring high molecular velocities, is escaping from the earth's atmosphere is not warranted, and, therefore, the conclusion that the vapor of water can not be retained by Mars is not warranted, at least under the conditions usually assumed for their atmospheres.

This paper would, however, not be complete without a reference to Dr. Stoney's reply[7] to the papers "On the Escape of Gases from Planetary Atmospheres According to the Kinetic Theory," by the writer, and "The Kinetic Theory of Planetary Atmospheres," by Dr. Bryan.

In his reply Dr. Stoney argues that the Boltzmann-Maxwell distribution will not account for the number of molecules attaining a velocity many times greater than the velocity of the mean square. Dr. Stoney concludes[8] that out of N free paths the actual number whose speed lies between v and $v + dv$ is

29 　　　　　　$N(\pi + \delta)\, dv$

where π is the probability function, which according to the Boltzmann-Maxwell law, is a function of v only, while δ is a function of the variables, v, h, n', θ, t, etc.

Where v is the speed; n, the number of molecules; n', the number of encounters; θ, the average duration of the free path; t, the average duration of an encounter; and where etc. stands for any other variable that might influence the value of δ.

Allowing the validity of this equation it seems from the nature of the functions δ and π that δ can not be many times greater than π. But even if δ could by some means attain to the value of $100\,\pi$ or $10,000\,\pi$ the permanency of an atmosphere of helium on the earth would not be materially affected, as will be evident by referring to Tables 2, 7, and 9. The fact that δ is a function of variables that may be either positive or negative would indicate that its value can not be large compared with the value of π, if indeed its value is not zero.

The value of Dr. Stoney's researches on the permanency of atmospheres must be determined more from the fact that they have opened up new fields of inquiry, and paved the way for the development of the kinetic theory of atmospheres, than from the specific result reached by the a priori method.

More recently M. E. Rogowsky[9] has discussed planetary atmospheres, but since he based his calculations on the results furnished by Dr. Stoney's memoir his conclusions, some of which are indeed very remarkable, must be modified in accordance with his note in Nature for July 3, 1902, i. e., in accordance with the results arrived at by the kinetic theory. In summing up these researches on the escape of gases from plane-

tary atmospheres and the kinetic theory of planetary atmospheres we conclude:

1. That helium forms a constituent though very small part of the earth's atmosphere,[10] and that according to the kinetic theory the earth will retain an atmosphere of helium at temperatures much higher than those that are known to prevail.

2. That the vapor of water will remain on the planet Mars at ordinary temperatures.

3. That according to the kinetic theory the moon, if it had a mean temperature of 0° C. would lose an atmosphere of nitrogen and oxygen.

4. That all the planets can retain atmospheres similar to the earth's atmosphere, and that the superior planets can retain atmospheres composed of gases much lighter than hydrogen.

CLIMATOLOGY OF COSTA RICA.

Communicated by H. PITTIER, Director, Physical Geographic Institute.

[For tables see the last page of this REVIEW preceding the charts.]

Notes on the weather.—On the Pacific slope the rain has been very scarce, the total amount for the month remaining in most cases inferior to the third part of the normal fall. As an immediate consequence, the coffee crop has been greatly diminished by premature ripening and by the havoc of several insect pests, the development of which has been favored by the prevailing drought. In San Jose the pressure has been about normal, the temperature slightly above the mean; rainfall 163 mm. against 241, normal; sky generally cloudy. On the Atlantic slope the rain has continued in excess of previous years, with the usual accompanying landslides and inundations.

Notes on earthquakes.—August 6, $0^h\ 10^m$ p. m., slight shock, E–W, intensity I, duration 2 seconds. August 11, $7^h\ 20^m$ p. m., slight shock, NE–SW, intensity II, duration 3 seconds. August 12, 8^h a. m., strong shock, E–W, intensity III, duration 6 seconds. August 13, $5^h\ 55^m$ a. m., tremors with several interruptions, total duration 8 seconds. August 16, $2^h\ 17^m$ a. m., several consecutive shocks, E–W, intensity III, duration 20 seconds. August 18, $11^h\ 31^m$ p. m., sensible shock, E–W, intensity III, duration 12 seconds.

HAWAIIAN CLIMATOLOGICAL DATA.

By CURTIS J. LYONS, Territorial Meteorologist.

GENERAL SUMMARY FOR AUGUST, 1902.

Honolulu.—Temperature mean for the month, 78.5°; normal, 77.7°; average daily maximum, 83.7°; average daily minimum, 74.2°; mean daily range, 9.5°; greatest daily range, 13°; least daily range, 5°; highest temperature, 86°; lowest, 72°.

Barometer average, 29.971; normal, 29.980; highest, 30.09, 29th; lowest, 29.86, 4th; greatest 24-hour change, that is, from any given hour on one day to the same hour on the next, .07; lows passed 4th and 24th; highs, 15th and 29th.

Relative humidity average, 70.5 per cent; normal, 68.5 per cent; mean dew-point, 67.3°; normal, 66°; mean absolute moisture, 7.32 grains per cubic foot; normal, 7.01 grains; dew on grass, 0.

Rainfall, 1.74 inches; normal, 1.97 inches; rain record days, 25; normal, 18; greatest rainfall in one day, 0.26 on the 14th; total at Luakaha, 9.08 inches; normal, 11.02 inches; total at Kapiolani Park, 0.42 inch; normal, 0.71 inch.

The artesian well level fell during the month from 33.40 to 33.10 feet above mean sea level. August 31, 1901, it stood at 33.30. The average daily mean sea level for the month was 9.78 feet, the assumed annual mean being 10.00 above datum. For August, 1901, it was 10.38. Trade wind days, 30 (3 of

[7] Astrophysical Journal, 11, pp. 251, 357, 1900.
[8] loc, cit. 22, pp. 363.
[9] Astrophysical Journal, November, 1901.

[10] Chemical News, 1895. Heinrich and Kayser. Nature, September 28, 1898. E. C. C. Baly. Nature, September 28, 1898. Ramsay & Travers. Nature, October 13, 1898. William Crookes. Nature, July 4, 1901. Prof. James Dewar.